

Technical Program Review  
**Nuclear Criticality Safety Program**  
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**Thermal Scattering Law Data Generation  
and  
Advanced Methods Development**

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# Acknowledgement

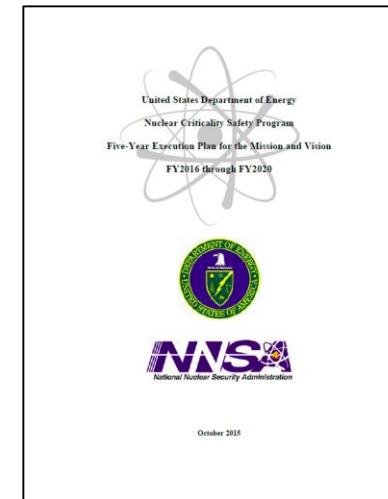
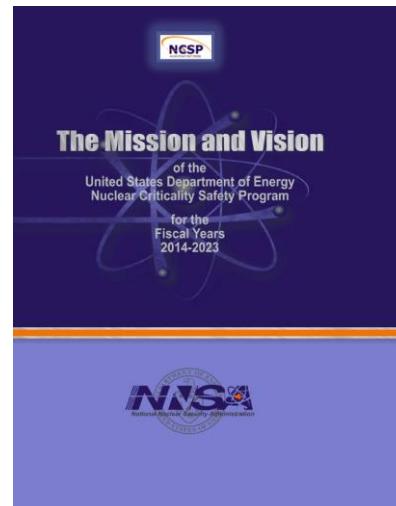
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- ❑ Funding by the NCSP and NNPP programs
- ❑ Collaboration with LLNL and BAPL
- ❑ Undergraduate and graduate students and postdocs at North Carolina State University



# Objectives

1. Generate TSL file 7 libraries (and related cross section files) for materials of interest to NCSP and NNPP
2. Develop the next generation tool for TSL data generation



## Appendix B Nuclear Data

Priority Needs \*/ Additional Needs

Thermal scattering (BeO, HF, D<sub>2</sub>O, SiO<sub>2</sub>, CH<sub>4</sub>, C<sub>2</sub>F<sub>6</sub>, C<sub>3</sub>O<sub>2</sub>H<sub>8</sub>, etc.), <sup>239</sup>Pu, Cr, <sup>237</sup>Np, Pb, <sup>55</sup>Mn, Ti, <sup>240</sup>Pu / <sup>233</sup>U, Th, Be, <sup>51</sup>V, Zr, F, K, Ca, Mo, Na, La

# Progress to Date

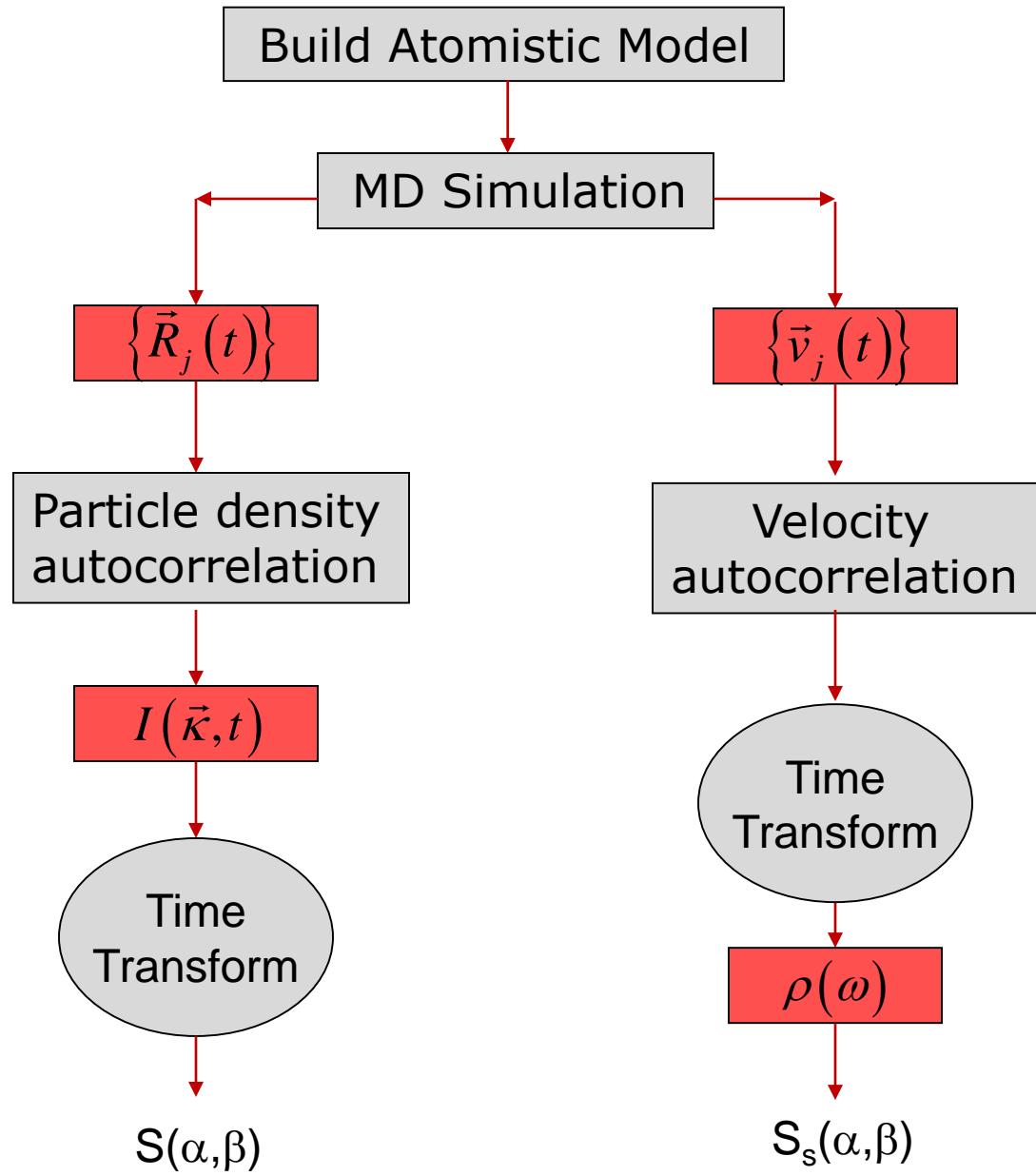
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Task	Expected completion	Actual completion	Comments
$\text{SiO}_2$	FY'14	FY'14	Contributed to NNDC
$\text{SiC}$	FY'14	FY'14	Contributed to NNDC
$(\text{C}_5\text{H}_8\text{O}_2)_n$	FY'15	FY'15	Contributed to NNDC
$(\text{CH}_2)_n$	FY'16	FY'15	File 7 ready for NNDC
Lubricant Oil	FY'17		On going
TSL Code Development	FY'16		<ol style="list-style-type: none"><li>1) Code flow and structure</li><li>2) Generalized coherent elastic scattering module</li><li>3) Generalized coherent inelastic scattering module</li></ol>

# Publications

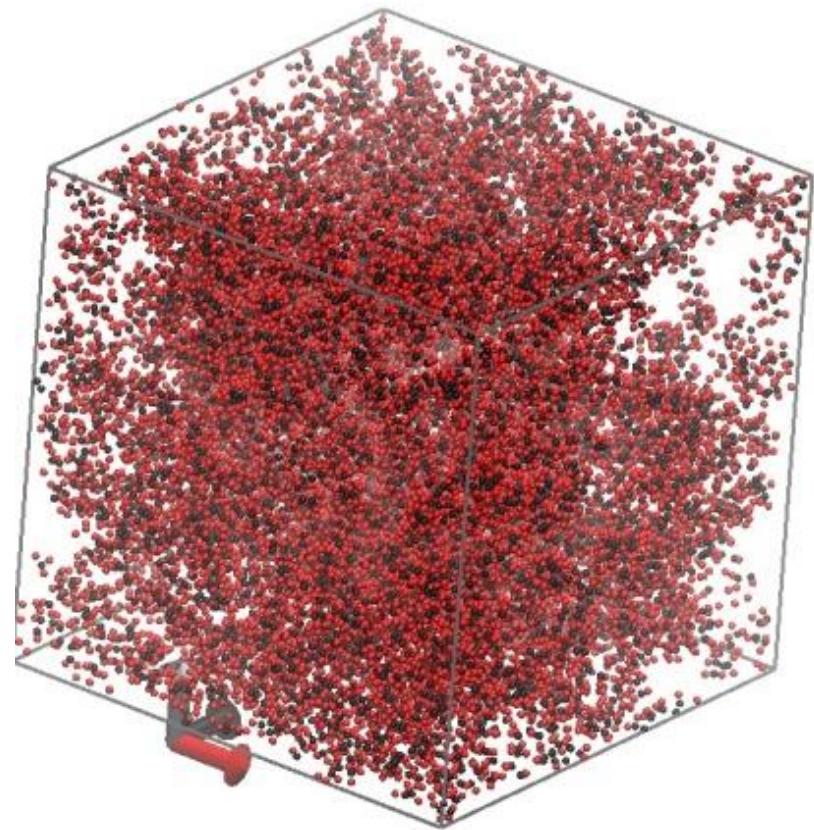
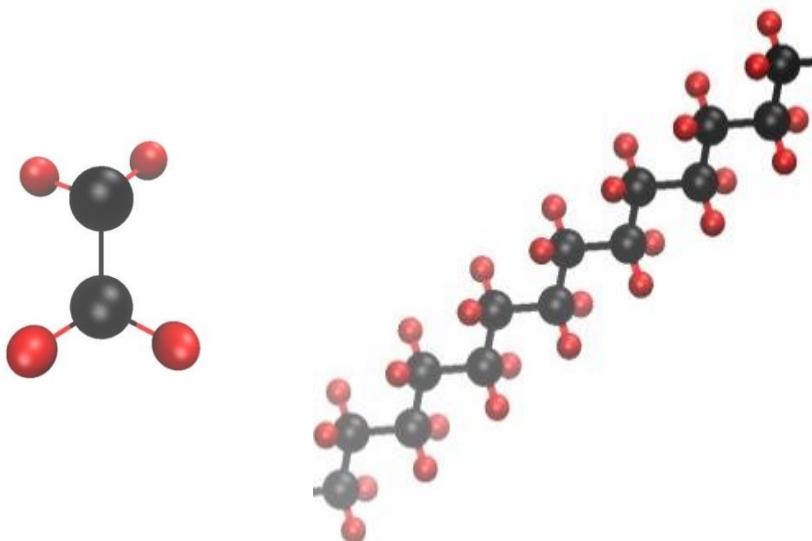
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1. Y. Zhu, A. I. Hawari, "Implementation of a Generalized Coherent Elastic Scattering Formulation for Thermal Neutron Scattering Analysis," Proceedings of International Conference on Nuclear Criticality Safety, Charlotte, NC, USA, 2015.
2. A. I. Hawari, A. Petersen, Y. Zhu, J. L. Wormald, D. P. Heinrichs, M. L. Zerkle, "Analysis of Thermal Neutron Scattering in Polymethyl Methacrylate (Lucite)," *Trans. American Nuclear Society*, 113, 2015.
3. Several abstracts (eventually papers) have been submitted to ND 2016



# Polyethylene MD Model

- ❑ Poly(ethylene) polymer
- ❑ 20 polymer chains
- ❑ 200 monomers ( $C_2H_4$ )  
long
- ❑ 24000 atoms



# MD Potential Function

$$E = E_{vdw} + E_Q + E_B + E_A + E_T$$

$$E_{vdw} = AR^{-12} - BR^{-6}$$

$$E_Q = CQ_iQ_j/\epsilon R_{ij}$$

$$E_B = \frac{1}{2} k_0(R - R_0)^2$$

$$E_A = \frac{1}{2} K_{IJK} (\cos\theta_{IJK} - \cos\theta)^2$$

$$E_T = E_{IJKL} = \frac{1}{2} V_{JK} \overline{\left\{ 1 - \cos[n_{JK}(\varphi - \varphi_{Jk}^0)] \right\}}$$

Dreiding force field

L-J (VDW) term:

**Fitted for the system to match  
the experimental density**

**> 0.9 g/cm<sup>3</sup>**

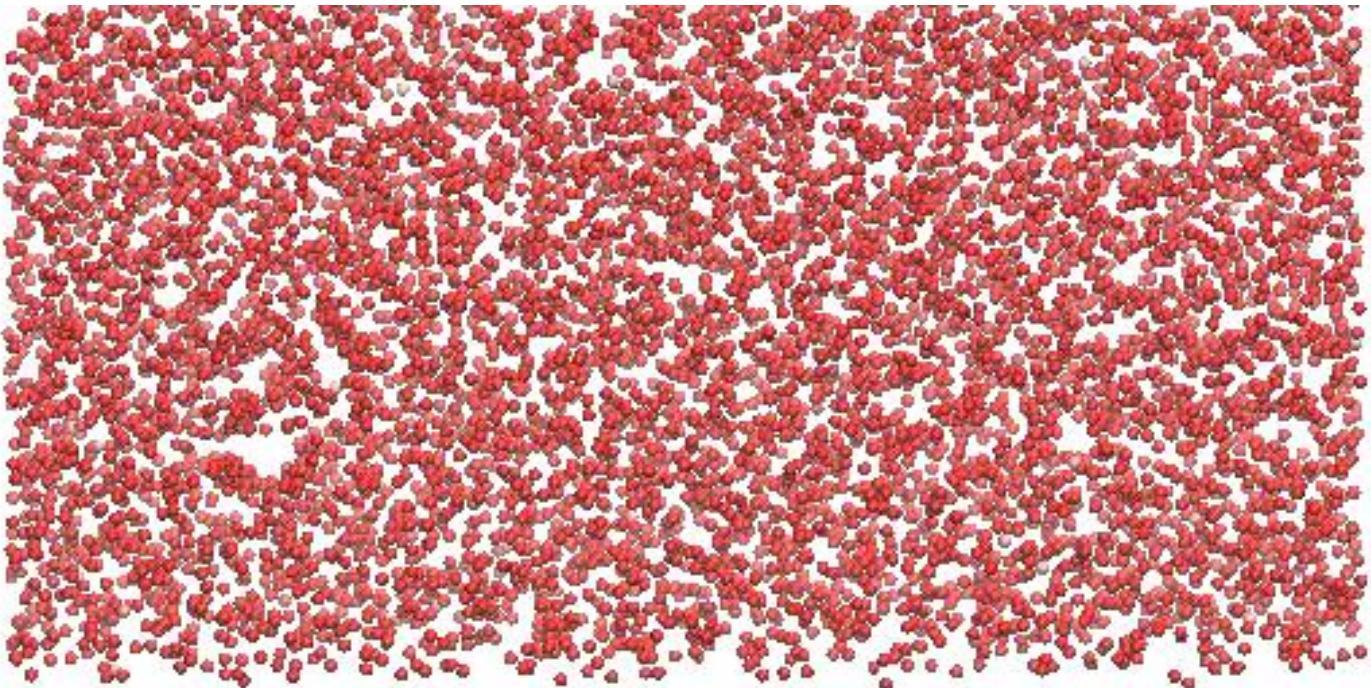
Dihedral barrier term:

**Fitted for the system to match  
the experimental  
amorphous/crystal composition**

**~ 80% crystal fraction**

# MD Simulation

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$T = 300 \text{ K}$     $P = 1 \text{ atm}$   
Iterate to achieve objectives

# Computational Approach

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$$C(t) = \left\langle \overrightarrow{v_j(0)} \cdot \overrightarrow{v_j(t)} \right\rangle = \frac{1}{N} \sum_{j=1}^N \overrightarrow{v_j(0)} \cdot \overrightarrow{v_j(t)}$$

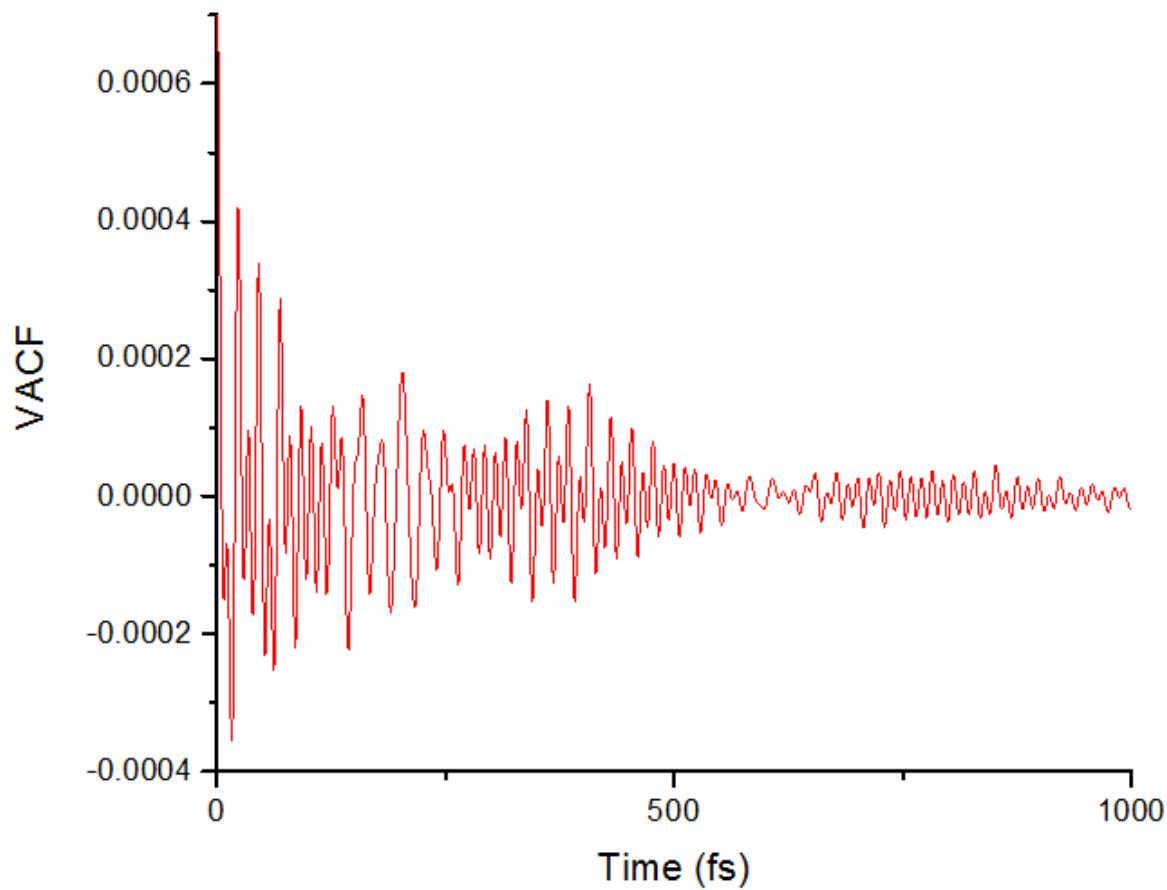
$$\rho(\omega) = \frac{M}{3\pi k_B T} \int_{-\infty}^{\infty} C(t) e^{-i\omega t} dt$$

Use NJOY/LEAPR       $S(\alpha, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\beta t} e^{-\gamma(t, \rho(\omega))} dt$

$$\left. \frac{d^2\sigma}{d\Omega dE} \right|_{\text{inelastic}} = \frac{\sigma}{2k_B T} \sqrt{\frac{E'}{E}} e^{-\frac{\beta}{2}} S(\alpha, \beta)$$

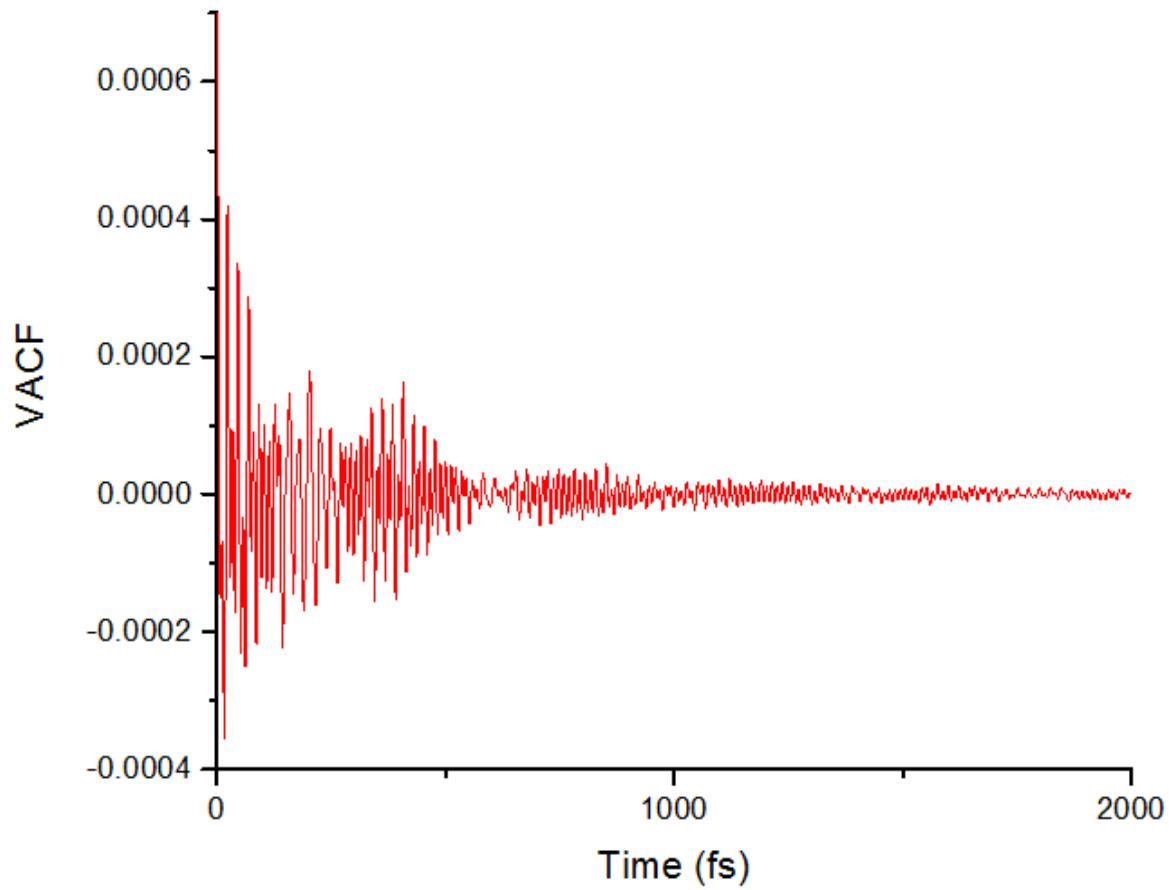
# Polyethylene VACF

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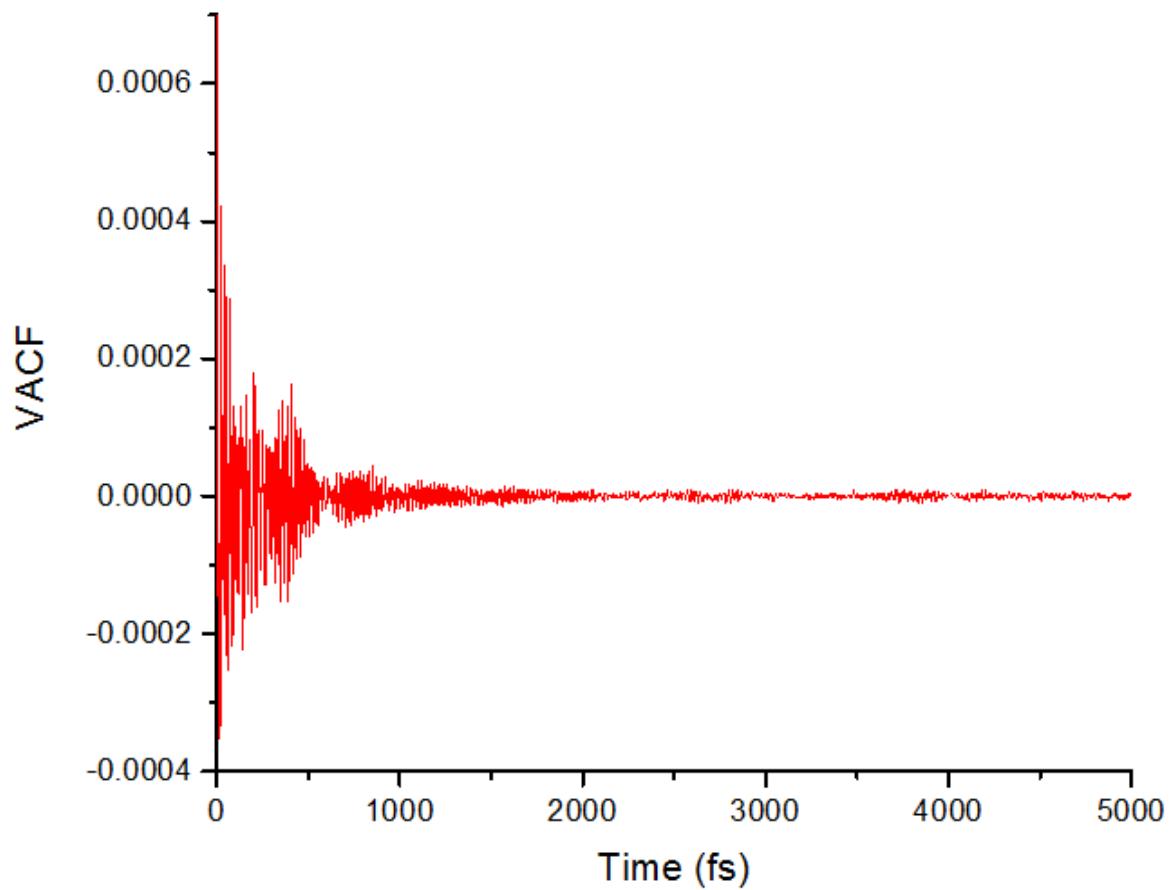
# Polyethylene VACF

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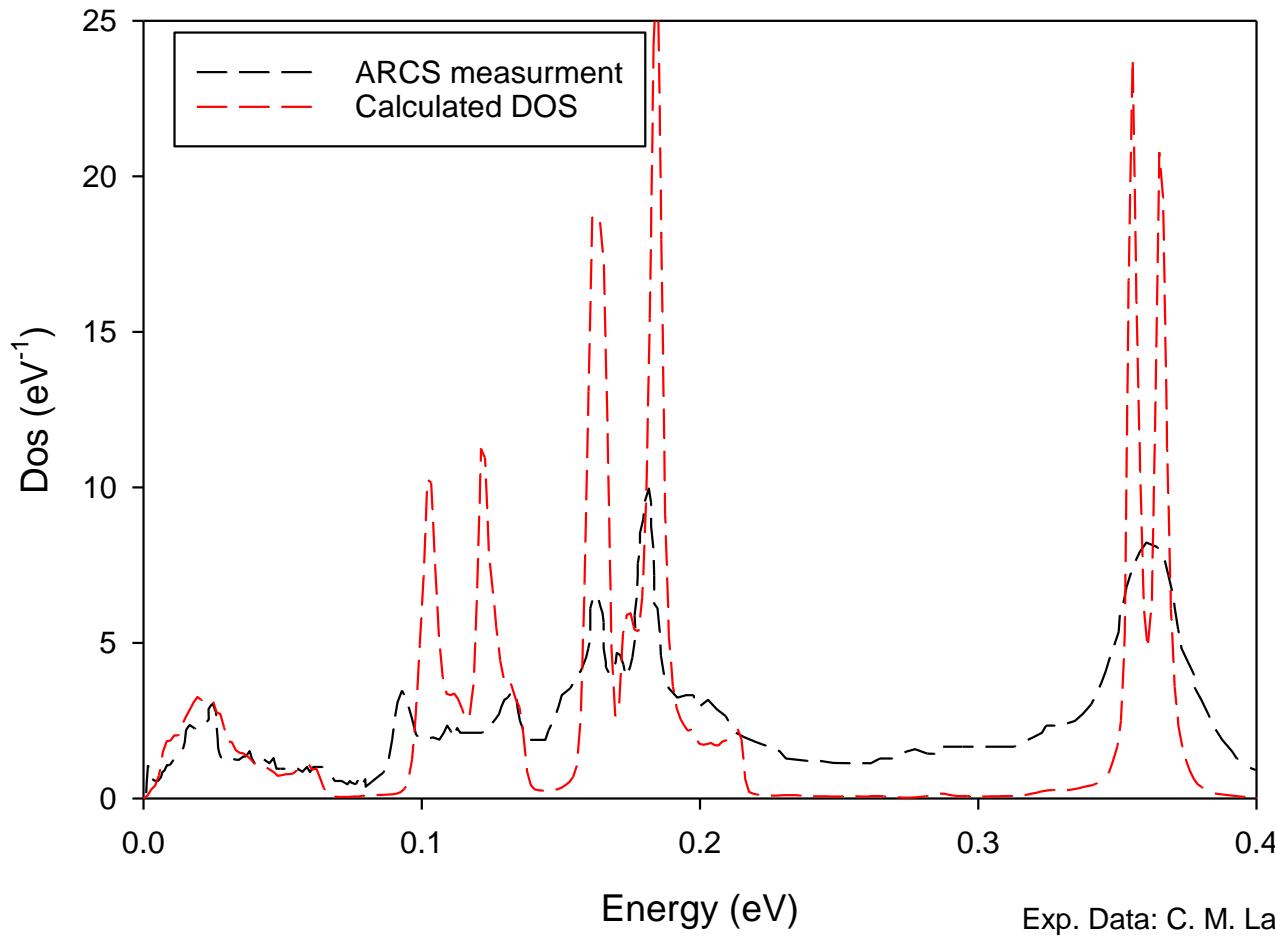


# Polyethylene VACF

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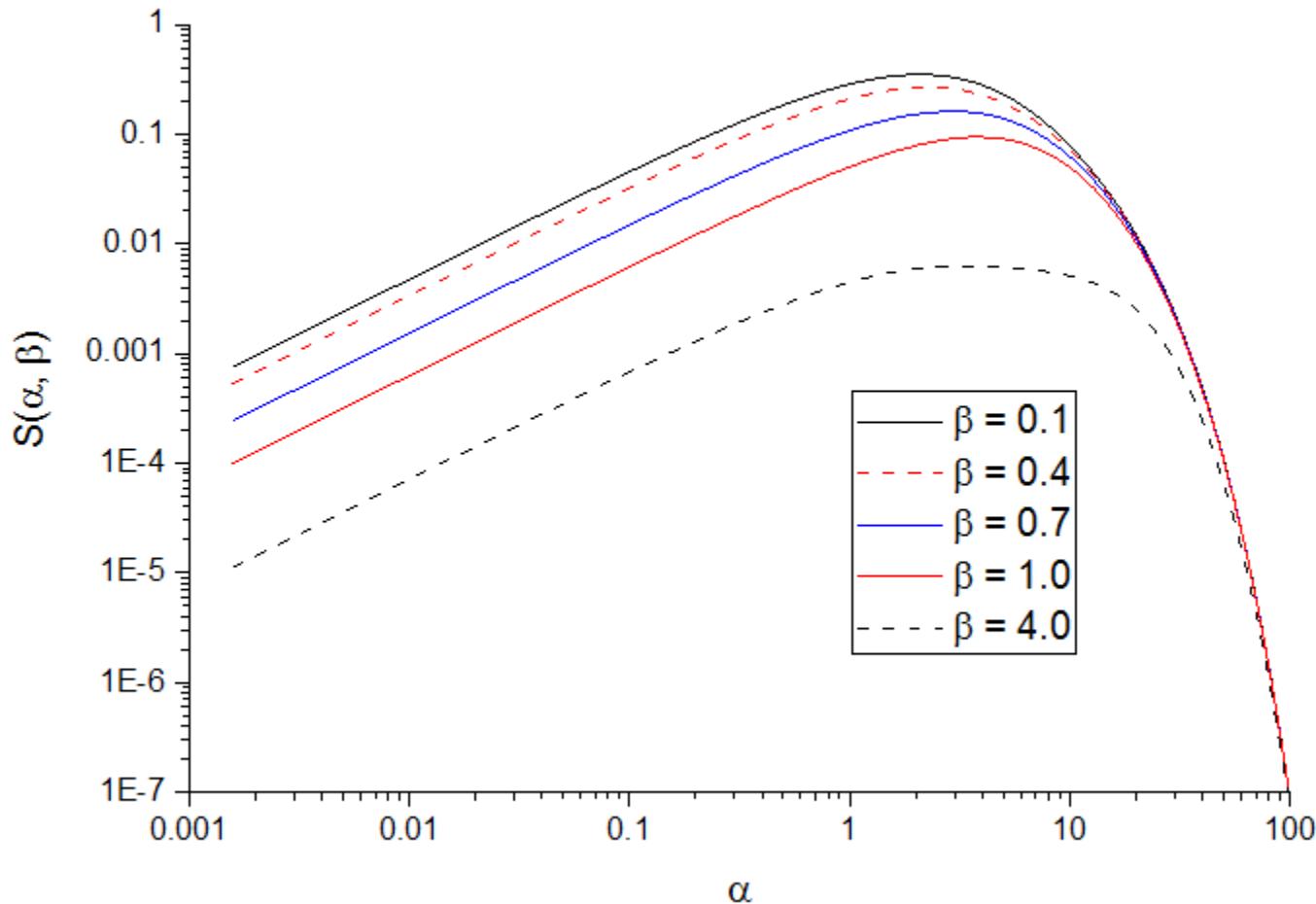


# Density of States

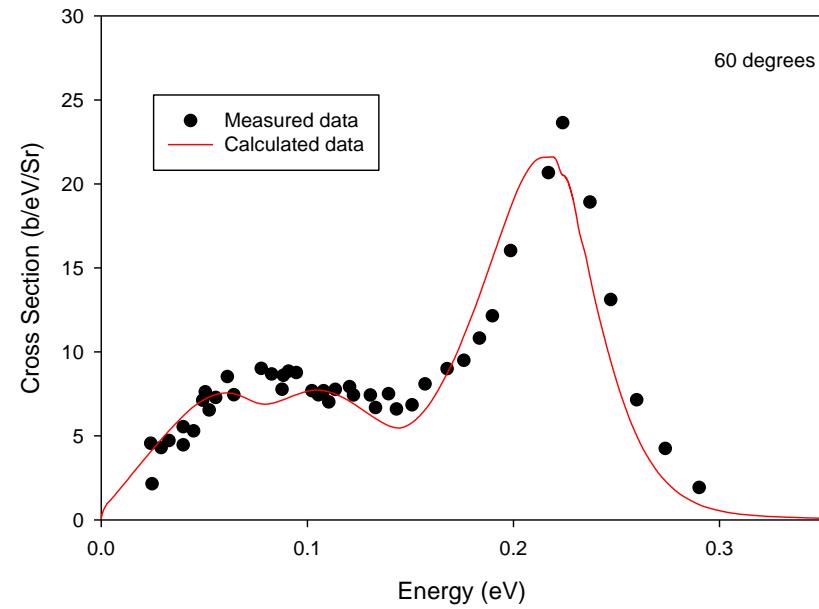
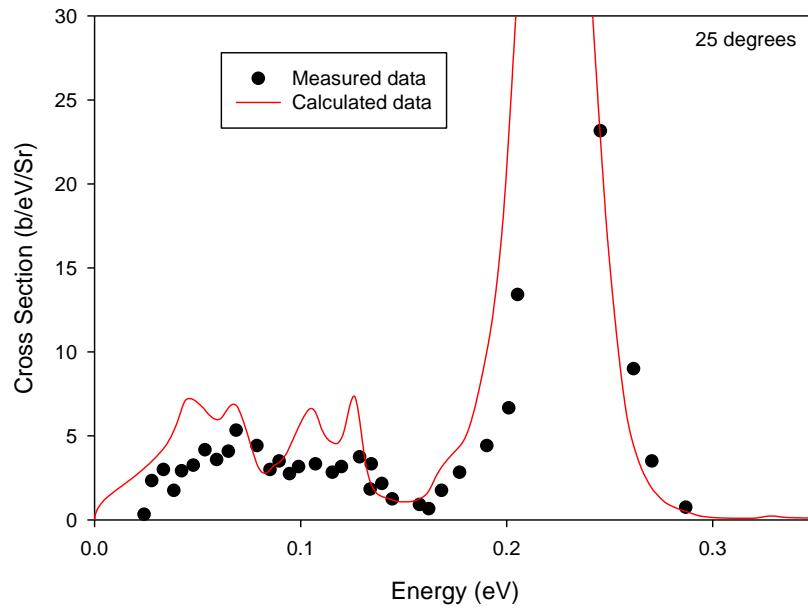


Exp. Data: C. M. Lavelle et al. , NIM A, 711,  
166 (2013).

# Thermal Scattering Law

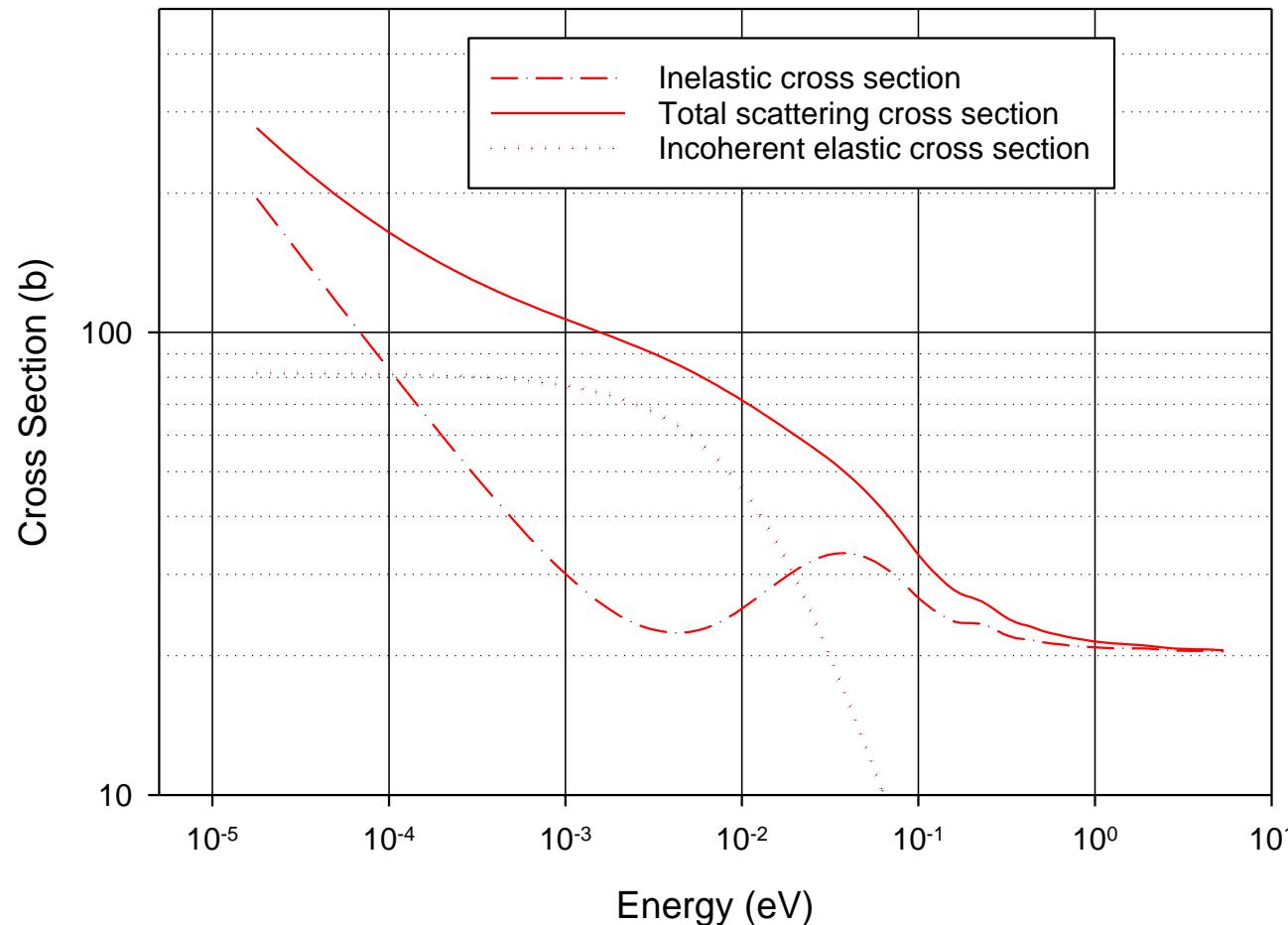


# Differential Cross Section

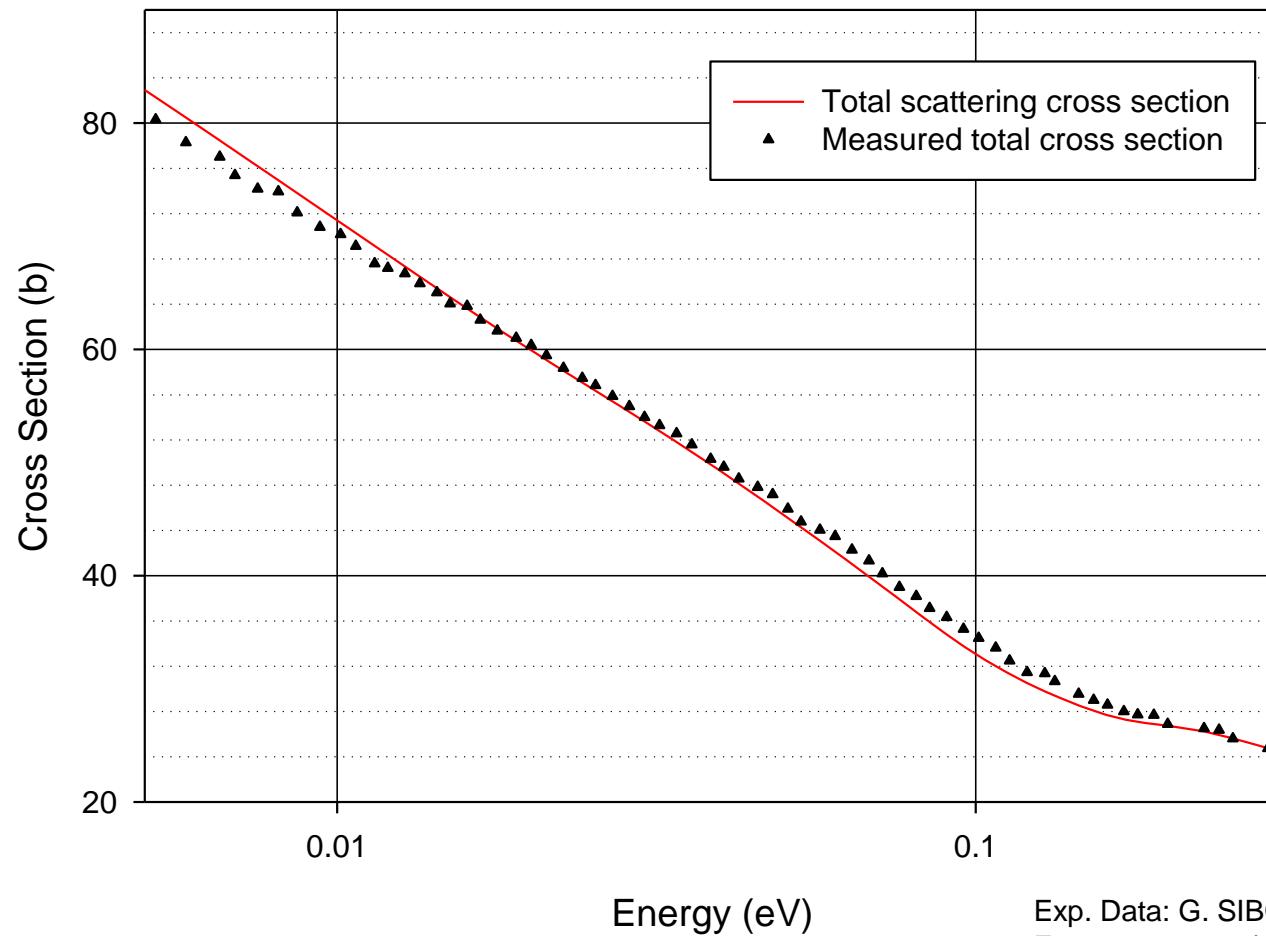


Exp. Data: R. E. Hill et al., NIM A, 538, 686  
(2005).

# Total Scattering Cross Section



# Total Scattering Cross Section



Exp. Data: G. SIBONA et al., Anna. Nucl. Energy, 18, 689 (1991).

# Polyethylene File 7

1.400000+2	9.991673-1	2	0	0	0	40	7	2	1
1.636270+2	0.000000+0	0	0	1	2	40	7	2	2
2	2					40	7	2	3
3.000000+2	2.716205+1	3.000000+2	2.716205+1			40	7	2	4
						40	7	099999	
1.400000+2	9.991673-1	0	1	0	0	40	7	4	1
0.000000+0	0.000000+0	0	0	12	1	40	7	4	2
4.087268+1	1.963720+2	9.991673-1	4.968212+0	0.000000+0	2.000000+0	40	7	4	3
1.000000+0	4.933334+0	1.189800+1	0.000000+0	0.000000+0	1.000000+0	40	7	4	4
0.000000+0	0.000000+0	0	0	1	200	40	7	4	5
200	4					40	7	4	6
3.000000+2	0.000000+0	0	0	1	100	40	7	4	7
100	4					40	7	4	8
1.572280-3	5.234086-4	1.795130-3	5.975359-4	2.049560-3	6.821499-4	40	7	4	9
2.340050-3	7.787328-4	2.671720-3	8.889769-4	3.050400-3	1.014807-3	40	7	4	10
3.482740-3	1.158416-3	3.976370-3	1.322317-3	4.539960-3	1.509358-3	40	7	4	11
5.183430-3	1.722796-3	5.918110-3	1.966338-3	6.756910-3	2.244203-3	40	7	4	12
7.714600-3	2.561198-3	8.808030-3	2.922795-3	1.005640-2	3.335201-3	40	7	4	13
1.148180-2	3.805532-3	1.310920-2	4.341788-3	1.496720-2	4.953085-3	40	7	4	14
1.708860-2	5.649809-3	1.951060-2	6.443655-3	2.227600-2	7.347967-3	40	7	4	15
2.543320-2	8.377686-3	2.903800-2	9.549856-3	3.315370-2	1.088356-2	40	7	4	16
3.785280-2	1.240035-2	4.321790-2	1.412435-2	4.934330-2	1.608259-2	40	7	4	17
5.633700-2	1.830534-2	6.432200-2	2.082618-2	7.343860-2	2.368227-2	40	7	4	18
8.384750-2	2.691473-2	9.573160-2	3.056845-2	1.093000-1	3.469236-2	40	7	4	19
1.247920-1	3.933937-2	1.424790-1	4.456546-2	1.626730-1	5.043017-2	40	7	4	20
1.857300-1	5.699506-2	2.120540-1	6.432177-2	2.421100-1	7.247176-2	40	7	4	21
2.764260-1	8.150194-2	3.156050-1	9.146205-2	3.603370-1	1.023904-1	40	7	4	22
4.114090-1	1.143079-1	4.697210-1	1.272112-1	5.362960-1	1.410634-1	40	7	4	23
6.123080-1	1.557866-1	6.990940-1	1.712490-1	7.981800-1	1.872544-1	40	7	4	24
9.113100-1	2.035309-1	1.040470+0	2.197207-1	1.187950+0	2.353766-1	40	7	4	25
1.356320+0	2.499540-1	1.548560+0	2.628284-1	1.768040+0	2.733076-1	40	7	4	26
2.018640+0	2.806712-1	2.304750+0	2.842184-1	2.631410+0	2.833377-1	40	7	4	27
3.004380+0	2.775855-1	3.430200+0	2.667702-1	3.916380+0	2.510240-1	40	7	4	28
4.471470+0	2.308484-1	5.105240+0	2.071131-1	5.828830+0	1.809961-1	40	7	4	29
6.654970+0	1.538625-1	7.598220+0	1.271018-1	8.675160+0	1.019553-1	40	7	4	30

# Lubricant Oil MD Model

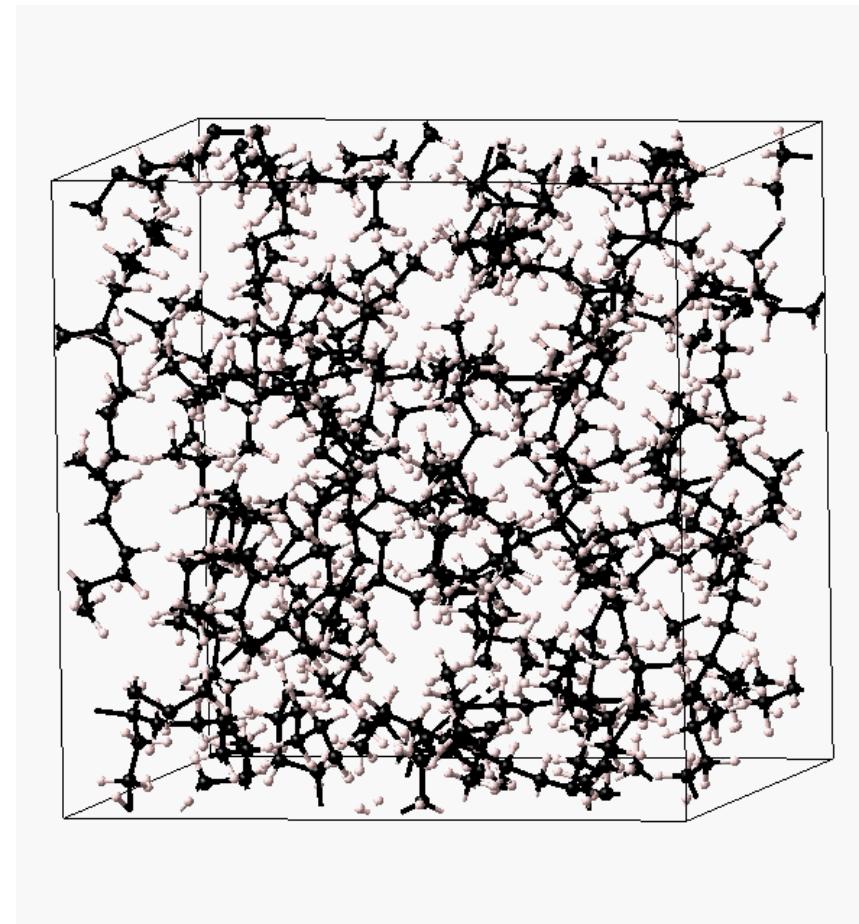
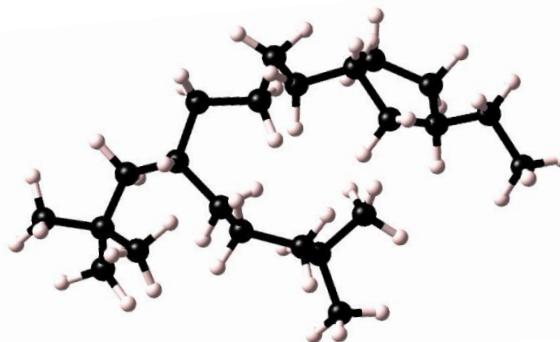
- ☐ Distilled and dewaxed heavy paraffinic mineral oil

- ☐ Example molecule

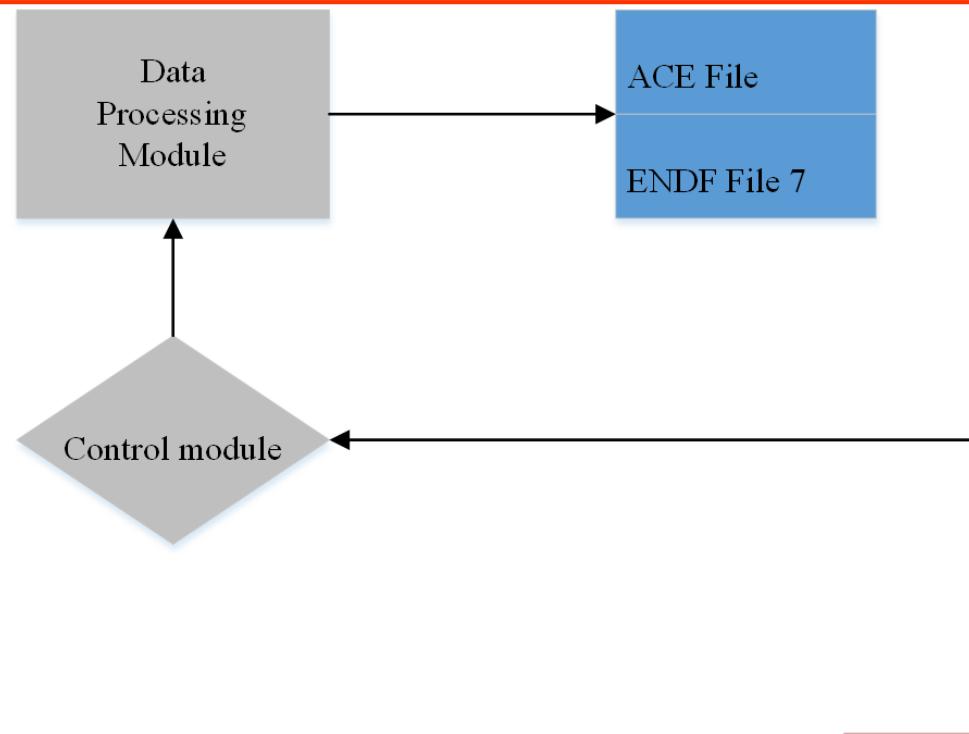
C<sub>23</sub>H<sub>48</sub>

- ☐ Test model 20 molecules

- ☐ Verification of physical properties is underway

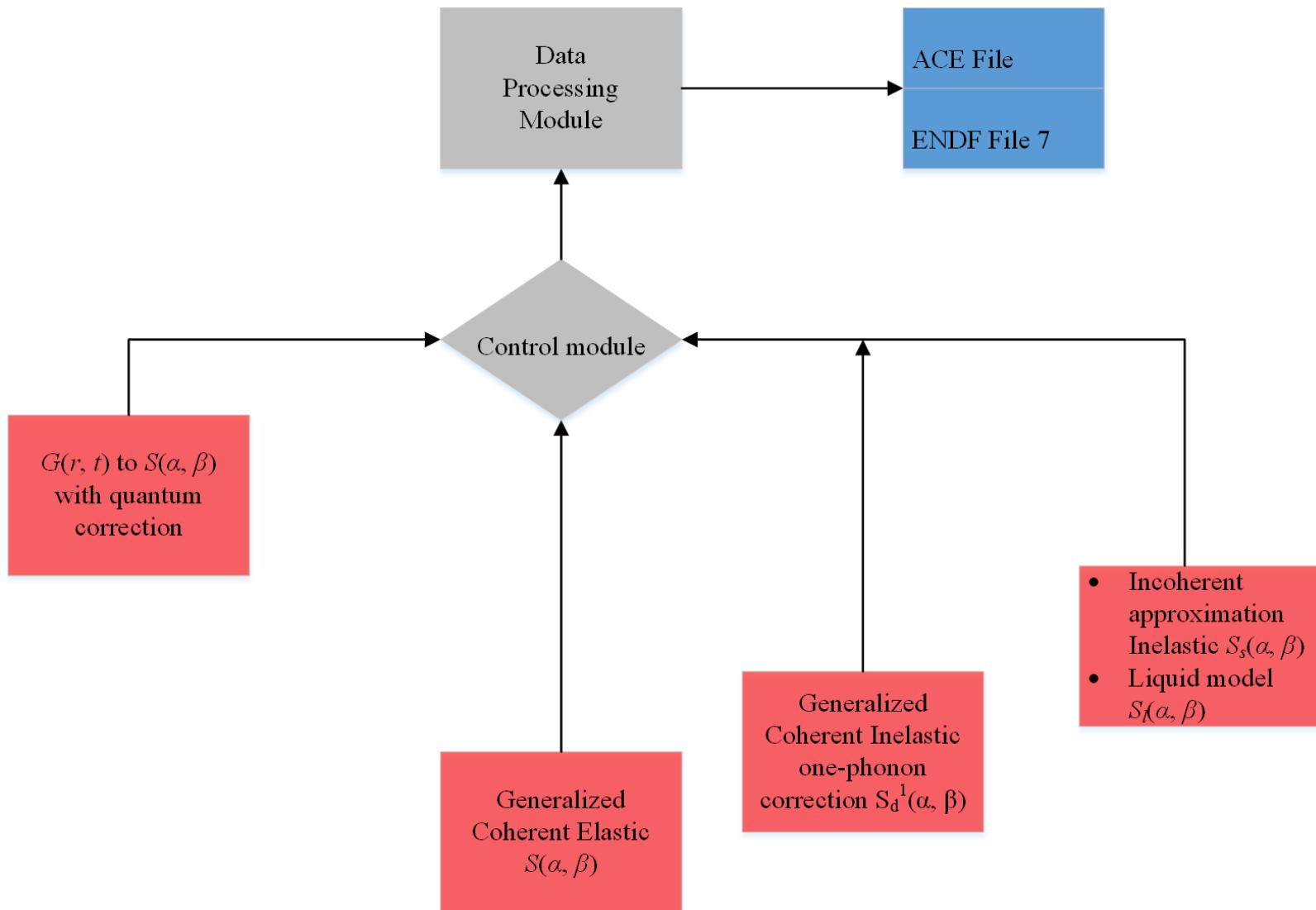


# Next Generation Code



- Incoherent approximation  
 $S_s(\alpha, \beta)$
- Limited coherent elastic scattering coverage
- Liquid model  
 $S_L(\alpha, \beta)$

# Next Generation Code



# Coherent Elastic Scattering

	Current	Generalized (NCSU)
<b>Supported structure</b>	<b>Hexagonal, FCC, BCC</b>	<b>Any crystal structure</b>
<b>Supported material</b>	<b>Graphite, beryllium, beryllium oxide, aluminum, lead, iron</b>	<b>Any material</b>
<b>Compound material</b>	<b>2 elements with ratio 1:1</b>	<b>Any number of elements with any ratio</b>
<b>Cubic Approximation</b>	<b>Yes</b>	<b>No</b>
<b>Atom sites approximation</b>	<b>Yes</b>	<b>No</b>
<b>Coherent Elastic Scattering Cross Section</b>	<b>Over Ewald Sphere</b>	<b>On every reciprocal space point</b>
<b>Need to modify source code if calculating other materials</b>	<b>Yes</b>	<b>No</b>

# Summary

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- Generation of TSL File 7 libraries is proceeding
  - Polyethylene completed in FY'15
  - Polymethyl Methacrylate (Lucite) contributed to NNDC in FY'15
  - New materials are initiated
- Work on “next generation” TSL generation code is ongoing. FY'16 tasks include
  - Defining code flow and structure
  - Integration of a generalized coherent elastic scattering module
  - Completing a generalized coherent inelastic scattering module